

SAMPLE

FURTHER MATHEMATICS
Teach Yourself Series
Topic 11: Measurement and Trigonometry

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Geometry

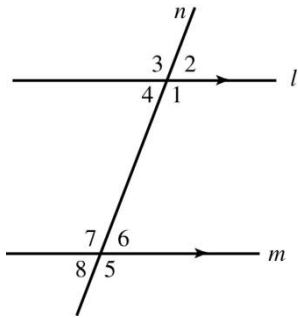
This area of study is important in many professions that use geometrical concepts and associated techniques- for example, architecture, navigation and art. It is a helpful tool to describe shapes of objects, directions on a car trip or design of a house.

Basic geometry

As it appears in Unit 2

- Point – marker for a location in space
- Line segment – two points connected
- Ray – line segment extended to infinity in one direction
- Line – line segment extended to infinity in both directions
- Angle – where two lines meet
- Parallel – two lines that never meet
- Perpendicular – two lines that cut each other at 90°
- Transversal – a line intersecting a pair of parallel lines
 - ◆ Corresponding angles are equal
 - ◆ Co-interior angles add up to 180°
 - ◆ Alternate angles are equal
- Vertically opposite angles – angles formed by two intersecting lines (are equal)
- Supplementary angles – add up to 180°
- Complementary angles – add up to 90°
- Polygon – closed figure made up of 3 or more line segments
- Sum of interior angles of a polygon - $S = (n - 2) \times 180^\circ$ where n is the no. of sides
- Regular polygon – polygon with equal sides/angles
- Interior angle of a regular n -sided polygon - $\frac{(n - 2) \times 180^\circ}{n}$
- Triangle – polygon with 3 sides
 - ❖ Sum of angles in a triangle is 180°
 - ❖ Circumcentre – where the perpendicular bisectors of a triangle meet
 - ❖ Centroid – point where the lines connecting each vertex with the midpoint of the opposite sides meet
 - ❖ Incentre – where the angle bisectors of each vertex meet
 - ❖ Scalene – triangle with no equal sides
 - ❖ Isosceles – triangle with two sides equal
 - ❖ Equilateral – triangle with all sides/angles equal

Example. For the following figure, if angle 3 is 105° answer **a** to **e** giving reasons



a. Find the value of angle 2

$$180^\circ - 105^\circ = 75^\circ \text{ (linear pair add up to } 180^\circ\text{)}$$

b. Find the value of angle 1

$$105^\circ \text{ (vertically opposite angles)}$$

c. Find the value of angle 6

$$75^\circ \text{ (angles 2 \& 6 are corresponding angles)}$$

d. Find the value of angle 8

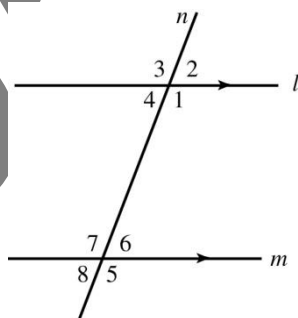
$$75^\circ \text{ (angles 6 \& 8 are vertically opposite angles)}$$

e. Find the value of angle 5

$$180^\circ - 75^\circ = 105^\circ \text{ (linear pair)}$$

Review Questions

1. Consider the following figure –



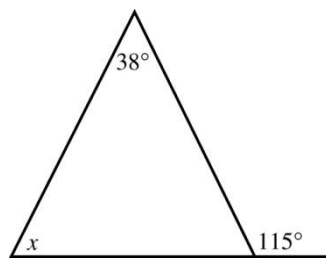
a. Write down pairs of angles which are vertically opposite

b. Write down pairs of angles which are co-interior

c. Write down pairs of angles which are alternate

d. Write down pairs of angles which are corresponding

2. From the following figure –



a. Find the angle marked x

b. Name the kind of triangle

3. Angles that add up to 180° are called

- A. Obtuse angles
- B. Reflex angles
- C. Vertically opposite angles
- D. Complementary angles
- E. Supplementary angles

Solutions to Review Questions

1.

a. 1 & 3; 2 & 4; 5 & 7; 6 & 8

b. 4 & 7; 1 & 6

c. 4 & 6; 1 & 7

d. 2 & 6; 3 & 7; 4 & 8; 1 & 5

2.

a. $115^\circ = x + 38^\circ$

$$x = 115^\circ - 38^\circ = 77^\circ$$

b. Scalene triangle

3. Answer: E

4. Answer: B

Explanation:

$$S = (n - 2) \times 180^\circ = (6 - 2) \times 180^\circ = 4 \times 180^\circ = 720^\circ$$

5.

a. $5 + 6 + 8 = 19\text{cm}$

b. $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{9.5(9.5-8)(9.5-6)(9.5-5)} = 15\text{cm}^2$

6.

a. Trapezium

b. $A = \frac{1}{2}(10+12) \times 7 = 77\text{cm}^2$

7.

a. $2(lw + wh + lh)$

b. $2(15 \times 9 + 9 \times 7 + 15 \times 7) = 606 \text{ m}^2 = 6060000 \text{ cm}^2$

c. $15 \times 9 \times 7 = 945 \text{ m}^3$

8.

a. Triangular prism

b. $TSA = \left(2 \times \frac{1}{2} \times 3 \times 4\right) + (5 \times 12) + (3 \times 12) + (4 \times 12) = 156 \text{ cm}^2$

c. $V = \frac{1}{2} \times 3 \times 4 \times 12 = 72 \text{ cm}^3$

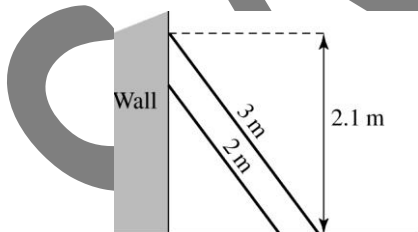
9. Answer: E

Explanation:

$$V = \frac{1}{2} \times \frac{4}{3} \times \pi \times 7.5^3 = 281.25\pi$$

10.

a.



b. $\frac{x}{2} = \frac{2.1}{3} \Rightarrow x = 1.4 \text{ m}$

11. Answer: D

Explanation:

Volume scale factor = 2 : 1

Length scale factor = $\sqrt[3]{2} : 1$

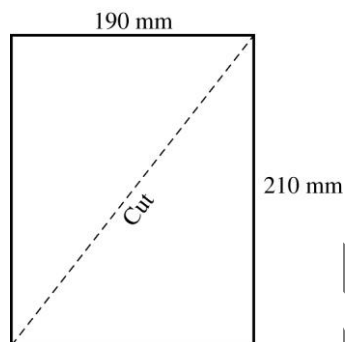
12. Answer: A

Explanation:

$$\frac{\text{volume of water}}{\text{volume of air}} = \frac{\frac{1}{3} \times \pi \left(\frac{r}{2}\right)^2 \left(\frac{1}{2}h\right)}{\frac{1}{3} \times \pi r^2 h - \frac{1}{3} \times \pi \left(\frac{r}{2}\right)^2 \left(\frac{1}{2}h\right)} = \frac{\frac{1}{8}}{\frac{7}{8}} = \frac{1}{7}$$

13.

a.



b.

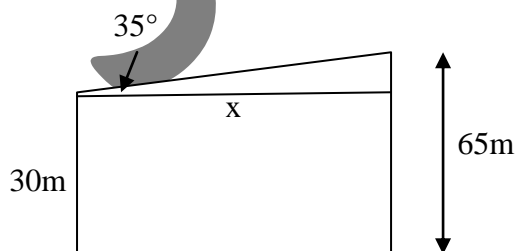
$$\begin{aligned} \text{Length of diagonal} &= \sqrt{210^2 + 190^2} \\ &\approx 283.196 \text{ mm} \end{aligned}$$

c.

$$\begin{aligned} \text{Length of width} &= \sqrt{490^2 - 190^2} \\ &\approx 451.664 \text{ mm} \end{aligned}$$

14.

a.



b. $\tan(35^\circ) = 35/x$ which gives $x = 50\text{m}$

c. 35° (it is the same as the angle of elevation)

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